BSSRDF Importance Sampling

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SOLIDANGLE  SAR ANGE  imageworks
Background

Most materials scatter light below the surface.
Background

Most materials scatter light below the surface
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Most materials scatter light below the surface
A BSSRDF* represents this process with a single formula (Jensen et al. [2001])

\[ S(x_i, \vec{\omega}_i, x_o, \vec{\omega}_o) \]

*Bidirectional Surface Scattering Distribution Function
At each shading point, an integral over the entire surface must be performed

\[ L_o(x_o, \bar{\omega}_o) = \int_A \int_{2\pi} S(x_i, \bar{\omega}_i; x_o, \bar{\omega}_o) L_i(x_i, \bar{\omega}_i)(\bar{\omega}_i \cdot \bar{n}_i) d\bar{\omega}_i dA(x_i) \]
We will assume the BSSRDF is radially symmetric and mostly a function of distance. Our talk focuses on *evaluating* the integral, not the shape of the BSSRDF.

\[
L_o(x_o, \vec{\omega}_o) = \int_A R(||x_i - x_o||) \int_{2\pi} L_i(x_i, \vec{\omega}_i)(\vec{\omega}_i \cdot \vec{n}_i)d\vec{\omega}_i dA(x_i)
\]
BSSRDFs are still actively being researched! [Habel et al., 2013], [d'Eon and Irving, 2011], ...

Simple alternatives to the more sophisticated models include:

Cubic: \( R(r) = \frac{10(R_{\text{max}} - r)^3}{\pi R_{\text{max}}^5} \)

Gaussian: \( R(r) = \frac{1}{2\pi\nu^2} e^{-r^2/2\nu^2} \)

\[
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\]
Previous Work - Point Clouds (Jensen and Buhler [2002])

- Distribute points across the mesh

Drawbacks
- Additional memory
- Requires a pre-pass
- Unfriendly to progressive rendering
- Point density ≈ mean free path
- Flickering artifacts
Distribute points across the mesh
Compute lighting at each sample
Previous Work - Point Clouds (Jensen and Buhler [2002])

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- Compute lighting at each sample
- Integrate by traversing data-structure

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Previous Work - Monte Carlo Methods

Three MC methods have been proposed
Christensen et al. [2012]

Multiresolution Radiosity Caching for Global Illumination in Movies
Previous Work - Monte Carlo Methods

Walter et al. [2012]
Bidirectional Lightcuts (Appendix B)
Previous Work - Monte Carlo Methods

Jensen et al. [2001]

A Practical Model for Subsurface Light Transport
Choose a point below the surface
Previous Work (1/3) - Christensen et al [2012]

Choose a point below the surface
Trace rays uniformly
Previous Work (1/3) - Christensen et al [2012]

Distribution depends on distance to surface
Samples have uniform probability but very different weights
Choose a radius with a pdf derived from $R(r)$
Choose a radius with a pdf derived from $R(r)$
Choose two random points on this sphere
Intersect ray segment with surface
Previous Work (2/3) - Walter et al [2012]

Requires 5 random numbers \((r, \theta_a, \phi_a, \theta_b, \phi_b)\)
Some rays do not produce any samples
Previous work (3/3) - Jensen et. al [2001]

Distribute points on disk around \( x_o \) according to \( R(r) \)
Previous work (3/3) - Jensen et. al [2001]

Optimal distribution for flat planes
Previous work (3/3) - Jensen et. al [2001]

Samples do not lie on the surface in general case
Hery [2003] proposed a simple top-down projection
Previous work (3/3) - Jensen et. al [2001]

Still not robust to all cases
Our approach

- Based on Jensen’s disk sampling approach
- Introduce a robust projection method
- Extend method to weighted sums of profiles
Our approach

Define $R_{\text{max}}$ to make search volume well defined

(Not a strict requirement, but keeps probe rays short)
Our approach

Distribute samples on disk above the surface according to $R(r)$
Our approach

Probe along normal direction, find all hits inside sphere
Our approach

Probe along normal direction, find all hits inside sphere

$x_0$
Our approach

In flat regions, $|\vec{n}_o \cdot \vec{n}_i| \rightarrow 1$ and $r_i \approx \|x_o - x_i\|$ so $w_i \approx 1$

$$w_i = \frac{R_d(\|x_o - x_i\|)}{\text{pdf}_{\text{disk}}(r_i)} \frac{1}{|\vec{n}_o \cdot \vec{n}_i|}$$
Our approach

In perpendicular regions: $|\vec{n}_o \cdot \vec{n}_i| \to 0$ and $r_i \neq \|x_o - x_i\|$ so $w_i \to \infty$

$$w_i = \frac{R_d(\|x_o - x_i\|)}{\text{pdf}_{\text{disk}}(r_i)} \frac{1}{|\vec{n}_o \cdot \vec{n}_i|}$$
Our approach

We solve this by probing along orthogonal directions as well.
Our approach - Axis MIS

Combine estimators with *multiple importance sampling* to keep the best of each
Our approach - Axis MIS

Some performance is lost in flat regions for more uniform variance overall
Axis MIS - Results

16 paths / pixel
Probe along N only
Axis MIS - Results

16 paths / pixel
Probe along U,V,N with MIS
Axis MIS - Results

64 paths / pixel
Probe along N only
Axis MIS - Results

64 paths / pixel
Probe along U,V,N with MIS
Implementation details

- Choice of axes is arbitrary, we found shading tangent frame works best
- Assign probabilities \((\vec{u}: \frac{1}{4}, \vec{v}: \frac{1}{4}, \vec{n}: \frac{1}{2})\)
- Re-use random number of axis selection for random radius
- Shade all hits encountered along the probe ray to handle thin regions
- Or stochastically choose one hit among all probe hits if splitting is not desirable
- Trace probe rays directly against the surface, bypassing scene hierarchy
Color MIS - Weighted Sums of BSSRDFs

Complex materials like skin are best expressed as weighted sums of simple profiles.
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Color MIS - Weighted Sums of BSSRDFs

MIS between profiles allows more efficient sampling than using the widest alone.
Results - Bonus Features

Fine details like bump are easy to preserve with a traced approach.
Results - Bonus Features

Diffuse BRDF

BSSRDF (no bump)

BSSRDF (with bump)
Results - Bonus Features

Albedo may be blurred along with illumination as well
Results - Bonus Features

Easy to blur across meshes to avoid seams
Results - Bonus Features

- Bi-directional extension is straightforward
- Create "virtual" vertex to continue path from
- Select probe hits stochastically if multiple are found
Results - Bonus Features

- Motion blur is always correct with traced approach
- Point clouds usually locked to a single instant
- Important for fast animation or large receivers
Production Results
Conclusion & Future Work

- Introduced a simple importance sampling method for BSSRDFs
- Only requires two random numbers and supports weighted sums of profiles
- Replaced a much more complex point cloud implementation
- Much easier for artists to use

- Perform a more detailed comparison with previous work
- Extend technique to work with non-radially symmetric profiles
Thanks for listening!

Questions?

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